Differential Euler Equations obtained from Causal Mathematical Logic

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Abstract Causal Mathematical Logic (CML) is used to obtain the Euler equations for the rotation of a rigid body directly from experimental measurements. The first ever use of the mathematical limit in the context of CML is also illustrated.

1 Introduction

Causal Mathematical Logic (CML) is a new fundamental theory of Physics [3-4]. The theory applies to information considered as a physical system with physical properties of its own. A general dynamical system can be analyzed by following the flow of information in relation with the dynamics. Observables can be derived by direct application of the theory. Scientists find observables and build theories. CML does too.

In this article, we assume that some experimental information describing the rotation of a rigid body has been obtained by an imaginary scientist who lived long before Euler and Newton and knows nothing about Newton's theory or Euler's equations. The measurements obtained by the scientist describe the causality involved in the motion of the rotating body, but other than that, the measured quantities have no meaning at all for the scientist.

This presentation is unusual. A traditional presentation would start by defining the concepts of angular velocity, torque, moments of inertia, and angular momentum, and then apply Newton's 2nd. law in vector form to obtain Euler's equations. Here, instead, we derive certain observables directly from measurement by considering only causal sequences described by ordered but otherwise completely meaningless (cause-effect) pairs. Once the observables are obtained, we argue that they make the experiment *understandable*, and therefore carry meaning. The ability of CML to create meaning in the same way we humans do is the major tenet of the causal theory.

The reason for the unusual presentation is to avoid a reversal of causality. A scientist researching a problem from scratch first solves the problem in his mind, then realizes that certain quantities that appeared in the equations have physical meaning, and finally presents a theory where the meaningful quantities are defined first, and then the equations are written in terms of the maningful quantities. Putting the meaning first is traditional for presentation purposes, but it entails a reversal of causality.

Causal models are always constructed with a certain *granularity*. A full causal model would have required to go all the way back to the Big Bang, which is not possible. Instead, an initial coarse model is constructed from "that what is known" at the time. Initial observables are computed, and examined. If more detail is needed, it is always possible to refine the model with additional information. A causal model can always be refined by adding more variables or more ordered pairs, obtained from more detailed measurements. More detailed observables can, then, be calculated and again examinded, and the process can be continued indefinitiely. A causal model is ideal as a permanent support model for a project under development. Compare these convenient advantages of causal models with the habitual stiffness of the more traditional models used in science.

Reference [4] is required reading. This article assumes that the reader is familiar with CML theory. Other recommended reading is [1-2].

2 The causal set

The imaginary scientist in the Introduction has measured the following 10 variables:

$$\text{measured variables: } A, B, C, D, E, F, G, H, I, J. \tag{1}$$

All variables are real numbers but the scientist does not know what they "mean", except for variable J, which is the time interval used between readings in the experiments. The scientist is also aware that the shorter the time step the more accurate the measurements are after many time steps. The scientist has expressed his observations as the following equations, where new intermediate real-valued variables have been introduced, and are indicated in lower case:

$$\begin{aligned}
a &= G \times J \\
b &= H \times J \\
c &= I \times J \\
d &= E - F \\
e &= F - D \\
f &= D - E \\
g &= d \times B \\
h &= e \times C \\
i &= f \times A \\
j &= g \times C \\
k &= h \times A \\
\ell &= i \times B \\
m &= j \times J \\
n &= k \times J \\
o &= e \times J \\
p &= a + m \\
q &= b + n \\
r &= c + o \\
s &= p/D \\
t &= q/E \\
u &= r/F
\end{aligned} \tag{2}$$

The lower-case variables in Eq. 2 are the elements of the causal set. Hence the causal set has $N_e = 21$ elements. The ordered pairs for the causal set are obtained by examining dependences in Eq. 2 that involve set elements, not just values that have been measured and are given. For example, the first equation $a = G \times J$ does not give rise to any pairs because both G and J are measured values. Expression $g = d \times B$ gives rise to ordered pair (d, g), expressing the fact that g depends on d, while B in the same equation is ignored. Expression $p = a + m$ gives rise to two pairs, (a, p) and (m, p), because p depends on both a and m. In all, the causal set has $N_r = 18$ relations.

Rigorously speaking, the algebraic operations represented by symbols $+, -, \times, /$ must also be cast in causal form. Doing that is not difficult, but would unnecessarily complicate this article. For our purposes, it will suffice to keep separate track of the operations involved in each ordered pair. These operations will reappear in the final equations. The resulting causal set is as follows:

$$S = \{a, b, c, d, e, f, g, h, i, j, k, \ell, m, n, o, p, q, r, s, t, u\} \tag{3}$$

$$\begin{aligned} \omega = \{ & d \prec g, e \prec h, f \prec i, g \prec j, h \prec k, i \prec \ell, j \prec m, k \prec n, e \prec o, \\ & a \prec p, m \prec p, b \prec q, n \prec q, c \prec r, o \prec r, p \prec s, q \prec t, r \prec u\} \end{aligned} \tag{4}$$

where S is the set and ω is the partial order. Symbol '$\prec$' means 'precedes'. When $N_e > N_r$, then the causal is partitioned into at least $N_c = N_e - N_r$ connected components. This is, already, the first result from CML theory, and it is telling us that the problem is partitioned into at least 3 connected components.

3 Calculating the block system

The original permutation of set S is given in Eq. 3 in an arbitrary orden, the alphabetical order of the names of the variables in this case. The value of the action functional for this permutation is 180 (dimensionless). The theory prescribes that invariant observables appear in the set of permutations having the least value of the action functional. These permutations form a grupoind, and the invariants appear as a group-theoretical block system. Hence, the next step is to find legal permutations of the causal set with the least value of the action. A total of 6 permutations were found by random search, and are shown in the following table. The least-action value for the permutations in the set is 42. Three connected components were obtained, shown with double lines in the table, and each connected component has a block system with 4 blocks, as indicated by single lines:

$$\left\| \begin{array}{cc|cc|cc|c||cc|cc|cc|c||cc|cc|cc|c} d & g & j & m & a & p & s & e & h & k & n & b & q & t & f & i & \ell & o & c & r & u \\ d & g & j & m & a & p & s & f & i & \ell & o & c & r & u & e & h & k & n & b & q & t \\ e & h & k & n & b & q & t & d & g & j & m & a & p & s & f & i & \ell & o & c & r & u \\ e & h & k & n & b & q & t & f & i & \ell & o & c & r & u & d & g & j & m & a & p & s \\ f & i & \ell & o & c & r & u & d & g & j & m & a & p & s & e & h & k & n & b & q & t \\ f & i & \ell & o & c & r & u & e & h & k & n & b & q & t & d & g & j & m & a & p & s \end{array} \right\| \tag{5}$$

The considerable reduction in the action, from 180 to 42, indicates strong invariants. Further analysis indicates that the three connected components have identical structure. In other words, they are 3 objects of the same class. And this is the second conclusion directly obtained from basic causal analysis: there is a single class, and there are 3 objects of that class. Names can now be assigned. The number 3 is the number of dimensions, and the 3 objects are the components of a vector. It is now necessary to reassemble the expressions from 2 into equations, and then rename the variables in a way that will make their meaning more obvious.

4 Assembling and understanding the equations

In this Section, we re-assemble the equations from Eq. 2 into the structure discovered by the block system of Eq. 5. We will do that for the first permutation of the left-most block in Eq. 5, which is ($dgjmaps$). In the process, we will also re-insert the algebraic signs from the original equations. This is easy to do. The result is:

$$\begin{aligned} d &= E - F \\ g &= (E - F)B \\ j &= (E - F)BC \\ m &= (E - F)BCJ \\ a &= GJ \\ p &= GJ + (E - F)BCJ \\ s &= [GJ + (E - F)BCJ]/D \end{aligned} \tag{6}$$

(7)

This same step must be completed for each one of the three sub-permutations in 5. When completed, the pattern followed by the expressions will become apparent. To enhance this effect, we first rename the 10 input variables and the 3 output variables as follows;

$$\begin{aligned} A &= \omega_1 \\ B &= \omega_2 \\ C &= \omega_3 \\ D &= I_1 \\ E &= I_2 \\ F &= I_3 \\ G &= M_1 \\ H &= M_2 \\ I &= M_3 \\ J &= \Delta t \\ s &= \Delta\omega_1 \\ t &= \Delta\omega_2 \\ u &= \Delta\omega_3 \end{aligned} \tag{8}$$

After renaming, the 3 final equations are:

$$\begin{aligned} \Delta\omega_1/\Delta t &= [M_1 + (I_2 - I_3)\omega_2\omega_3]/I_1 \\ \Delta\omega_2/\Delta t &= [M_2 + (I_3 - I_1)\omega_3\omega_1]/I_2 \\ \Delta\omega_3/\Delta t &= [M_3 + (I_1 - I_2)\omega_1\omega_2]/I_3 \end{aligned} \tag{9}$$

The final step is to take the mathematical limit for small steps: $\Delta t \to 0$. Using the classical notation $\dot{\omega}$ for the time derivative $d\omega/dt$, the Euler equations are obtained in final form:

$$\begin{aligned} I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 &= M_1 \\ I_2\dot{\omega}_2 + (I_1 - I_3)\omega_3\omega_1 &= M_2 \\ I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 &= M_3 \end{aligned} \tag{10}$$

(11)

where I_1, I_2, I_3 are the moments of inertia, M_1, M_2, M_3 are the components of the applied torque, and $\omega_1, \omega_2, \omega_3$ are the components of the angular velocity.

5 References

[1] S. Pissanetzky. "A new universal model of computation and its contribution to learning, intelligence, parallelism, ontologies, refactoring, and the sharing of resources." Int. J. of Computational Intelligence, **5**, 143-173 (Aug. 2009).

[2] S. Pissanetzky. "Coupled dynamics in host-guest complex systems duplicates emergent behavior in the brain." Proc. Int. Conf on Computation and Software Engineering, Singapore, **69**, 927-935 (Aug. 2010).

[3] S. Pissanetzky. (2011a). Emergence and Self-organization in Partially Ordered Sets. Complexity 17(2): 1938. Note: the partially ordered sets mentioned in the title are actually causal sets.

[4] S. Pissanetzky. "Reasoning with Computer Code: a new Mathematical Logic, Special issue on Self-Programming, K. R. Thórisson, E. Nivel and R. Sanz (eds.), Journal of Artificial General Intelligence, Special Issue on Self-programming, Vol. 3, issue 3, pages 11-42, December 2012. DOI: 10.2478/v10229-011-0020-6. Open access.